TROPICAL CUBIC SURFACES

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Arthur Cayley and George Salmon proved in the 1840s that every smooth cubic surface in $\mathbb{P}^3$ contains exactly 27 lines. Since the early development of tropical geometry, two natural problems were to understand whether the same statement holds for the tropicalization of cubic surfaces and to classify the combinatorial positions of their tropical lines. The answer to the first turned out to be false, as smooth tropical surfaces might contain families of tropical lines. Moreover, classifying positions of tropical lines reveals some computational challenges due to the massive number of combinatorial types of tropical cubic surfaces.

Alternatively, Ren, Shaw and Sturmfels considered the intrinsic tropicalization of very affine surfaces obtained from cubic surfaces by removing the 27 lines. They identified two generic types of such tropical surfaces characterized by their structure at infinity, which is an arrangement of 27 trees with 10 leaves.

In this talk we will tell this tropical story and compare the two approaches highlighting the computational aspects. We will then introduce an octanomial model for cubic surfaces. This new normal form is well suited for p-adic geometry, as it reveals the intrinsic del Pezzo combinatorics of the 27 trees in the tropicalization. The talk is based on joint work with Emre Sertöz and Bernd Sturmfels.